

INFRARED BEHAVIOUR OF GLUON AND GHOST PROPAGATORS IN QCD IN THE LANDAU GAUGE

P.O. FEDOSENKO¹

¹*Kyiv National Taras Shevchenko University, Physics Department, Ukraine**

Non-perturbative formalism of the generalized effective action is used for deriving the Schwinger-Dyson equations. In order to clear the domain of integration in the functional integral from gauge copies a restriction to the Gribov horizon due to Zwanziger is implemented. In this approach an asymptotic behaviour of gluon propagator and propagator of the Faddeev-Popov ghosts at small momenta is studied. Such a behaviour is obtained as a result of solving coupled Schwinger-Dyson equations in zeroth and first-order approximation. The qualitative agreement of these results with the ones obtained before is demonstrated and quantitative difference in some coefficients is found.

I. INTRODUCTION

It is widely believed that Quantum Chromodynamics (QCD) is the theory which describes the strong interactions. Physical phenomena at large momenta transfers are very well described by perturbation theory as the coupling becomes small. Asymptotic freedom allows to picture high energy quarks and gluons as weakly interacting particles. This picture, however, starts to break down at intermediate momenta and is surely inadequate at energies below a few hundred MeV. At such scales the interaction is strong enough to invalidate perturbation theory and one has to employ completely different methods to study non-perturbative phenomena like confinement or chiral symmetry breaking. One such method is the study of the Schwinger-Dyson equations (SDEs).

Schwinger-Dyson equations are the equations of motion for Green's functions of a field theory. In QCD the study of the non-perturbative behaviour of these functions is interesting for several reasons. One of such reasons is that confinement mechanism like the Gribov-Zwanziger scenario [1, 2, 14] is related to the infrared behaviour of the ghost and gluon propagators.

Many attempts have been made to understand the gluon propagator behaviour through SDEs. In the late seventies Mandelstam initiated the study of the gluon SDE in the Landau gauge [3]. Neglecting the ghost fields contribution and imposing cancellations of certain terms in the gluon polarization tensor, he found a highly singular gluon propagator in the infrared. This enhanced gluon propagator was appraised for many years in the literature, firstly because it provided a simple picture of quark confinement, since it is possible to derive from it an inter-quark potential that rises linearly with the separation, and secondly because the gluon propagator, which is singular as $1/q^4$, has enough strength to support dynamical chiral symmetry breaking. However, these re-

sults are discarded by simulations of QCD on the lattice [5] where it is shown that the gluon propagator is probably infrared finite.

Later, infrared finite solutions were also found in the Schwinger-Dyson approach [4]. Considering coupled gluon and ghost SDEs it was shown that gluon propagator is suppressed and ghost propagator is enhanced in the infrared region. Brief overview of the results on the infrared properties of the gluon and ghost propagators from SDEs existing in today's literature is made in Section II

In the main part of this work (Section 3) the formalism of the generalized effective action proposed by Cornwall, Jackiw and Tomboulis (CJT) [6] is investigated. The effective action, which depends on the gluon and ghost propagators and the Gribov mass parameter is obtained. In Section 4 we construct SDEs for the gluon and ghost propagators as well as for the Gribov parameter in one-loop approximation. Notice, that the SDEs obtained in our work are not exactly the same ones, discussed in literature [7, 8, 9, 10]. Let us clarify this point.

The starting point in most papers is a full system of SDEs. It is a coupled, nonlinear system of equations, which contains dressed propagators as well as dressed vertex functions on the right-hand side. In order to obtain a closed system of equations it is necessary to specify suitable approximations for these equations. There are many truncation schemes, developed for this purpose, for example replacing dressed vertices by their tree-level expressions. Furthermore, in this formulation, Gribov's prescription of cutting off the functional integral at the Gribov horizon does not change the Schwinger-Dyson equations, but rather resolves an ambiguity in the solution of these equations.

On the other hand, in our presentation we restrict ourselves to two-loop approximation of the effective action and obtain a closed system of SDEs without any other truncation schemes. These three equations are derived as a result of equating to zero the functional derivatives of the generalized effective action with respect to the gluon and ghost propagators and the Gribov parameter.

*Email address: fedosenko@univ.kiev.ua

Restriction to the (first) Gribov region appears explicitly via modification of the free gluon propagator, which enters the gluon SDE explicitly and, therefore, two other SDEs are affected via the dressed gluon propagator too.

In Section 5 the zeroth-order analysis of the derived SDEs is carried out. It is noteworthy that at this order we get the results for the gluon propagator and horizon condition similar to those previously obtained by Gribov. It is shown that the ghost propagator obtained at this order and Gribov's one indicate the same qualitative behaviour $1/q^4$ in the infrared. However they differ in some coefficient.

The full analysis of the derived one-loop SDEs is developed in Section VI. The infrared critical exponents are obtained as well as the numerical coefficients for the propagators. In the last Section (7) the discussion of obtained results is presented.

II. NOWADAYS STATE OF AFFAIRS

The infrared behaviour of the Landau gauge gluon and ghost propagators is an interesting and hot subject. Up to now there are two competitive viewpoints on this problem.

Let us represent the gluon and ghost propagators, respectively, in the form:

$$G_{\mu\nu}^{ab}(q) = C_G \frac{\tilde{G}(q^2)}{q^2} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \delta^{ab}, \quad (1)$$

$$\mathcal{D}^{ab}(q) = C_D \frac{\tilde{D}(q^2)}{q^2} \delta^{ab}, \quad (2)$$

where C_G and C_D are dimensionless constants. We will seek for the solutions of the gluon and ghost dressing functions in the form of simple power laws

$$\tilde{G}(q^2) \propto (q^2)^{-\alpha_G} \quad (3)$$

and

$$\tilde{D}(q^2) \propto (q^2)^{-\alpha_D}, \quad (4)$$

respectively. Coefficients α_G and α_D are called infrared critical exponents or anomalous dimensions.

The first point of view is as follows. Using the gluon propagator SDE as well as the ghost propagator SDE it is claimed that $\alpha_G + 2\alpha_D = -(4-d)/2$, [7, 8], i.e. for the dimension of space-time $d = 4$ one gets $\alpha_G + 2\alpha_D = 0$. The interesting point here is that self-consistency forces an interrelation of the exponents such that they depend on one parameter $k = \alpha_D = -\alpha_G/2$ only. The value of the exponent k is in the range $0.5 < k \leq 1$, [7, 8, 9, 10], depending on details of the truncation of the set of SDEs.

Some of the authors report on the ambiguous result [7]:

$$\alpha_G = -2, \alpha_D = 1 \quad \text{and} \quad \alpha_G \approx -1.1906, \alpha_D \approx 0.5953. \quad (5)$$

It is worth noticing, that the Kugo-Ojima confinement criterion [11] in terms of infrared exponents requires $k > 0$, which means that the ghost propagator should be more singular and the gluon less singular than a simple pole. On the other hand, Gribov-Zwanziger confinement scenario [1, 12, 13, 14] requires the same condition for the ghost propagator, but it needs $k > 0.5$ for the gluon dressing function. Thus, available results for the exponents are in good agreement with these two widespread confinement scenarios.

However there is another point of view on this problem. Some of the authors [15] report that $\alpha_G + 2\alpha_D \neq 0$ and correct value of α_D is close to zero while α_G is close to 1. Thus they claim that in the infrared limit the ghost dressing function is finite and differs from zero.

The aim of the present paper is independently determine the infrared critical exponents.

III. CORNWALL-JACKIW-TOMBOULIS FORMALISM

We start to consider this problem with making use of the formalism of the generalized effective action proposed by J. Cornwall, R. Jackiw and E. Tomboulis [6]. This generalization of the effective action, $\Gamma(\phi, G)$, depends not only on ϕ – a possible expectation value of the quantum field $\Phi(x)$ – but also on $G(x, y)$ – a possible expectation value of a bilocal operator $T\Phi(x)\Phi(y)$ – time ordered product of two fields. Physical solutions require

$$\frac{\delta\Gamma(\phi, G)}{\delta\phi(x)} = 0, \quad \frac{\delta\Gamma(\phi, G)}{\delta G(x, y)} = 0. \quad (6)$$

In our derivation we identify $\Phi(x)$ with the gluon field, so $G(x, y)$ is the gluon propagator. We introduce two more quantities – the ghost propagator $\mathcal{D}(x, y)$ and the so-called Gribov parameter γ . Parameter γ , known also as the Gribov mass, characterizes the restriction of the domain of integration in the functional integral to the so-called Gribov horizon. This restriction is necessary due to the existence of the Gribov copies, which imply that the Landau condition, $\partial_\mu A_\mu = 0$, does not uniquely fix the gauge and equivalent gauge copies still exist in the domain of integration of the functional integral [1, 17].

We start from the generating functional for Green's functions of nonlocal, composite fields:

$$Z(J, K) = e^{\frac{i}{\hbar} W(J, K)} = \int D\Phi e^{\frac{i}{\hbar} S_{eff}}, \quad (7)$$

$$S_{eff} = -\frac{1}{4} \int d^4x (F_{\mu\nu}^a(x))^2 + \int d^4x d^4y \bar{C}^a(x) J^{ab}(x, y) C^b(y) + \\ + \frac{1}{2} \int d^4x d^4y A_\mu^a(x) K_{\mu\nu}^{ab}(x, y) A_\nu^b(y) + S_z(\Phi(x)), \quad (8)$$

where

$$S_z(\Phi(x)) = \int d^4x \{ g\gamma^2 f^{abc} (A_\mu^a \varphi_\mu^{bc} + A_\mu^a \bar{\varphi}_\mu^{bc}) - \lambda^a (\partial_\mu A_\mu^a) - \\ - \bar{C}^a \partial_\mu (D_\mu C)^a - \bar{\varphi}_\mu^{ac} \partial_\nu (D_\nu \varphi_\mu)^{ac} + \bar{\omega}_\mu^{ac} \partial_\nu (D_\nu \omega_\mu)^{ac} + \\ + g(\partial_\nu \bar{\omega}_\mu^{ac}) f^{abm} (D_\nu C)^b \varphi_\mu^{mc} + 4\gamma^4 (N^2 - 1) \} \quad (9)$$

is the expression, which characterizes Zwanziger's formulation of the Gribov horizon [14]. It is BRST-invariant together with the first term of (8). Functional differential $D\Phi$ labels the product of differentials of all fields entering the integrand (effective action (8)):

$$D\Phi \equiv dAdCd\bar{C}d\lambda d\varphi d\bar{\varphi} d\omega d\bar{\omega}. \quad (10)$$

Let us clarify the notations above. Here A is the gluon field with components A_μ^a . Fields C and \bar{C} are the Faddeev-Popov ghosts which have components C^a and \bar{C}^a . Multiplier λ^a enforces the constraint $\partial_\mu A_\mu^a = 0$, characteristic of the Landau gauge. Fields $\varphi \equiv -\frac{1}{\sqrt{2}}(\varphi_1 + i\varphi_2)$ and $\bar{\varphi} \equiv \frac{1}{\sqrt{2}}(\varphi_1 - i\varphi_2)$ are the pair of boson complex fields with components $\varphi_i^a \equiv \varphi_\mu^{ac} \equiv f^{abc} \varphi_\mu^b$ and f^{abc} are the structure constants of the $SU(N)$ gauge group. Here we use the single index $i \equiv (\mu, c)$ for the pair of mute indices, and i takes on $f = d(N^2 - 1)$ values ($d = 4$ stands for the dimension of space). The fields ω and $\bar{\omega}$ are the Grassmann fields, which have the same components as φ and $\bar{\varphi}$. The vector indices μ, ν take on d values, while the color indices a, b refer to the adjoint representation of the $SU(N)$ group and take on $N^2 - 1$ values.

We introduced two bilocal sources $K_{\mu\nu}^{ab}(x, y)$ and $J^{ab}(x, y)$ for the gluon field A_μ^a and for the ghosts \bar{C}^a, C^a respectively.

Let $G_{\mu\nu}^{ab}(x, y)$ be a possible expectation value of $TA_\mu^a(x)A_\nu^b(y)$ and $\mathcal{D}^{ab}(x, y)$ stands for a possible expectation value of $TC^a(x)\bar{C}^b(y)$:

$$G_{\mu\nu}^{ab}(x, y) \equiv \langle TA_\mu^a(x)A_\nu^b(y) \rangle, \quad (11)$$

$$\mathcal{D}^{ab}(x, y) \equiv \langle TC^a(x)\bar{C}^b(y) \rangle. \quad (12)$$

From eqs. (7) and (8) it follows that

$$\frac{\delta W(J, K)}{\delta K_{\mu\nu}^{ab}(x, y)} = \frac{1}{2} \hbar G_{\mu\nu}^{ab}(x, y), \quad (13)$$

$$\frac{\delta W(J, K)}{\delta J^{ab}(x, y)} = -\hbar \mathcal{D}^{ba}(y, x). \quad (14)$$

The effective action $\Gamma(G, \mathcal{D})$ is defined as a Legendre transform of $W(J, K)$:

$$\Gamma(G, \mathcal{D}) = W(J, K) - \frac{\hbar}{2} \int d^4x d^4y G_{\mu\nu}^{ab}(x, y) K_{\mu\nu}^{ab}(x, y) + \\ + \hbar \int d^4x d^4y \mathcal{D}^{ba}(y, x) J^{ab}(x, y). \quad (15)$$

We eliminate K and J in favor of G and \mathcal{D} using the next obvious relations:

$$K_{\mu\nu}^{ab}(x, y) = -\frac{2}{\hbar} \frac{\delta \Gamma(G, \mathcal{D})}{\delta G_{\mu\nu}^{ab}(x, y)} \quad (16)$$

and

$$J^{ab}(x, y) = \frac{1}{\hbar} \frac{\delta \Gamma(G, \mathcal{D})}{\delta \mathcal{D}^{ba}(y, x)}. \quad (17)$$

Then according to (15), we have

$$\exp \frac{i}{\hbar} \Gamma(G, \mathcal{D}) = \exp \frac{i}{\hbar} W(J, K) \times \\ \times \exp \left\{ -\frac{i}{2} \int d^4x d^4y G_{\mu\nu}^{ab}(x, y) K_{\mu\nu}^{ab}(x, y) + \right. \\ \left. + i \int d^4x d^4y \mathcal{D}^{ba}(y, x) J^{ab}(x, y) \right\}. \quad (18)$$

In order to shorten our notations we will omit dependence on coordinates and the corresponding integration where it is obvious, keeping it in mind. Taking into account (7), (8) together with (16) and (17) we have

$$\exp \frac{i}{\hbar} \Gamma(G, \mathcal{D}) = \int D\Phi \exp \frac{i}{\hbar} \left\{ -\frac{1}{4} (F_{\mu\nu}^a)^2 - \frac{1}{\hbar} A_\mu^a \frac{\delta \Gamma}{\delta G_{\mu\nu}^{ab}} A_\nu^b + \right. \\ \left. + \frac{1}{\hbar} \bar{C}^a \frac{\delta \Gamma}{\delta \mathcal{D}^{ba}} C^b + G_{\mu\nu}^{ab} \frac{\delta \Gamma}{\delta G_{\mu\nu}^{ab}} + \mathcal{D}^{ba} \frac{\delta \Gamma}{\delta \mathcal{D}^{ba}} + S_z(\Phi) \right\}. \quad (19)$$

After straightforward computations, for the first exponent term in the last expression one gets

$$-\frac{1}{4} (F_{\mu\nu}^a)^2 = \frac{1}{2} A_\nu^a (iD_F^{-1})_{\mu\nu}^{ab} A_\mu^b - g f^{abc} ((\partial_\mu A_\nu^a) A_\mu^b A_\nu^c) - \\ - \frac{1}{4} g^2 f^{abc} f^{apr} A_\mu^b A_\nu^c A_\mu^p A_\nu^r. \quad (20)$$

Here D_F is a free gluon propagator:

$$(iD_F^{-1})_{\mu\nu}^{ab} \equiv (\partial^2 g_{\mu\nu} - \partial_\mu \partial_\nu) \delta^{ab}. \quad (21)$$

The series expansion for $\Gamma(G, \mathcal{D})$ is

$$\Gamma(G, \mathcal{D}) = \frac{1}{2} i \hbar Tr \ln(G^{-1})_{\mu\nu}^{ab} + \frac{1}{2} i \hbar Tr \left[(\tilde{D}_F^{-1})_{\mu\nu}^{ab} G_{\mu\nu}^{ab} \right] + \Gamma_2(G) - i \hbar Tr \ln(\mathcal{D}^{-1})^{ab} - i \hbar Tr \left[(S_F^{-1})^{ab} \mathcal{D}^{ba} \right] + \Gamma_2(\mathcal{D}) + \text{Const}, \quad (22)$$

where S_F is a free ghost propagator and \tilde{D}_F is a free gluon propagator, modified with respect to the Zwanziger term:

$$\tilde{D}_F^{-1} = D_F^{-1} + 2g^2 \gamma^4 N (i\partial^2)^{-1}. \quad (23)$$

The quantity Γ_2 is given by all two-particle irreducible vacuum graphs and it is of order \hbar^2 , because the number of loops corresponds to powers of \hbar . The quantities $\Gamma_2(G)$ and $\Gamma_2(\mathcal{D})$ are determined by the expansion by powers of \hbar :

$$\Gamma_2(G) = \hbar^2 \Gamma_2^{(1)}(G) + O(\hbar^3), \quad (24)$$

$$\Gamma_2(\mathcal{D}) = \hbar^2 \Gamma_2^{(1)}(\mathcal{D}) + O(\hbar^3). \quad (25)$$

From the expressions (19), (20) the series expansions (22), (24) and (25), using the notation

$$\Gamma'(G, \mathcal{D}) \equiv -i \hbar Tr \ln(\mathcal{D}^{-1})^{ab} + \hbar^2 \Gamma_2^{(1)}(\mathcal{D}) - \hbar^2 \mathcal{D}^{ba} \frac{\delta \Gamma_2^{(1)}(\mathcal{D})}{\delta \mathcal{D}^{ba}} + \frac{1}{2} i \hbar Tr \ln(G^{-1})^{ab} + \hbar^2 \Gamma_2^{(1)}(G) - \hbar^2 G_{\mu\nu}^{ab} \frac{\delta \Gamma_2^{(1)}(G)}{\delta G_{\mu\nu}^{ab}}, \quad (26)$$

one obtains apart from the constant $\frac{i}{2} \hbar Tr 1$:

$$\begin{aligned} \Gamma'(G, \mathcal{D}) = & -i \hbar \ln \int D\Phi \exp \frac{i}{\hbar} \{ 4\gamma^4 (N^2 - 1)V + \\ & + \frac{i}{2} A_\nu^a (G^{-1} - 2g^2 \gamma^4 N (i\partial^2)^{-1})_{\mu\nu}^{ab} A_\mu^b + i \bar{C}^a (\mathcal{D}^{-1} - \\ & - S_F^{-1})^{ab} C^b + g \gamma^2 f^{abc} (A_\mu^a \varphi_\mu^{bc} + A_\mu^a \bar{\varphi}_\mu^{bc}) - \lambda^a (\partial_\mu A_\mu^a) + \\ & + \bar{\Theta}_i \partial^2 \Theta_i - g f^{abc} (\partial_\mu A_\nu^a) A_\mu^b A_\nu^c - g f^{acb} \bar{C}^a \partial_\mu A_\mu^c C^b + \\ & + g f^{acb} \bar{\omega}_i^a \partial_\mu A_\mu^c \omega_i^b - g f^{acb} \bar{\varphi}_i^a \partial_\mu A_\mu^c \varphi_i^b + g (\partial_\nu \bar{\omega}_\mu^{ac}) f^{abm} \times \\ & \times (\partial_\nu C)^b \varphi_\mu^{mc} - \frac{1}{4} g^2 f^{abc} f^{apr} A_\mu^b A_\nu^c A_\mu^p A_\nu^r - \hbar A_\mu^a \frac{\delta \Gamma_2^{(1)}(G)}{\delta G_{\mu\nu}^{ab}} A_\nu^b + \\ & + \hbar \bar{C}^a \frac{\delta \Gamma_2^{(1)}(\mathcal{D})}{\delta \mathcal{D}^{ba}} C^b + g^2 f^{abm} (\partial_\nu \bar{\omega}_\mu^{ac}) A_\nu^\lambda C^{\lambda b} \varphi_\mu^{mc}. \quad (27) \end{aligned}$$

Here V is the space-time volume of the system and we denote the sum of three terms of similar form by means of one term:

$$\bar{\Theta}_i \partial^2 \Theta_i \equiv \left(-\bar{C}^a \partial^2 C^a - \bar{\varphi}_\mu^{ac} \partial^2 \varphi_\mu^{ac} + \bar{\omega}_\mu^{ac} \partial^2 \omega_\mu^{ac} \right). \quad (28)$$

Thus we have the expression (27) for the effective action, and we have to evaluate the functional integral. But this expression includes not only terms, quadratic on the fields, which we are able to integrate, but also cubic ones and terms of fourth order on the fields. In order to compute it, one has to expand cubic and higher-order terms by powers of some small parameter in usual way. Following Cornwall et al., we will use \hbar (to be precise, we use not \hbar , but $\sqrt{\hbar}$) as such a small parameter, and this procedure is known as loop-expansion. We will retain only terms up to $\hbar = (\sqrt{\hbar})^2$, this corresponds to two-loop approximation.

In order to select terms by powers \hbar one has to change the scale of all 8 fields:

$$\Phi(x) \rightarrow \sqrt{\hbar} \Phi(x). \quad (29)$$

After rescaling the fields one can represent the series expansion by powers of small parameter \hbar for (27) in such a way:

$$\begin{aligned} \Gamma'(G, \mathcal{D}) = & -i \hbar \ln \int D\Phi \hbar^4 e^{\left\{ \frac{i}{\hbar} 4\gamma^4 (N^2 - 1)V + S^q \right\}} \times \\ & \times \left(1 + \sqrt{\hbar} S^{(\sqrt{\hbar})} + \hbar S^{(\hbar)} + \frac{1}{2} \left(\sqrt{\hbar} S^{(\sqrt{\hbar})} + \hbar S^{(\hbar)} \right)^2 + o(\hbar) \right), \quad (30) \end{aligned}$$

where S^q denotes quadratic terms on the fields:

$$\begin{aligned} S^q \equiv & -\frac{1}{2} A_\nu^a (G^{-1} - 2g^2 \gamma^4 N (i\partial^2)^{-1})_{\mu\nu}^{ab} A_\mu^b - \bar{C}^a (\mathcal{D}^{-1} - S_F^{-1})^{ab} C^b + \\ & + i g \gamma^2 f^{abc} (A_\mu^a \varphi_\mu^{bc} + A_\mu^a \bar{\varphi}_\mu^{bc}) - i \lambda^a (\partial_\mu A_\mu^a) + \bar{\Theta}_i \partial^2 \Theta_i; \quad (31) \end{aligned}$$

The term $S^{(\sqrt{\hbar})}$ denotes the cubic ones:

$$\begin{aligned} S^{(\sqrt{\hbar})} \equiv & i(-g f^{abc} (\partial_\mu A_\nu^a) A_\mu^b A_\nu^c - g f^{acb} \bar{C}^a \partial_\mu A_\mu^c C^b + \\ & + g f^{acb} \bar{\omega}_i^a \partial_\mu A_\mu^c \omega_i^b - g f^{acb} \bar{\varphi}_i^a \partial_\mu A_\mu^c \varphi_i^b + \\ & + g (\partial_\nu \bar{\omega}_\mu^{ac}) f^{abm} (\partial_\nu C)^b \varphi_\mu^{mc}), \quad (32) \end{aligned}$$

and

$$\begin{aligned} S^{(\hbar)} \equiv & i \left(-\frac{1}{4} g^2 f^{abc} f^{apr} A_\mu^b A_\nu^c A_\mu^p A_\nu^r - A_\mu^a \frac{\delta \Gamma_2(G)}{\delta G_{\mu\nu}^{ab}} A_\nu^b + \right. \\ & \left. + \bar{C}^a \frac{\delta \Gamma_2(\mathcal{D})}{\delta \mathcal{D}^{ba}} C^b + g^2 f^{abm} (\partial_\nu \bar{\omega}_\mu^{ac}) A_\nu^\lambda C^{\lambda b} \varphi_\mu^{mc} \right) \quad (33) \end{aligned}$$

denotes the terms of the fourth order on the fields.

Carrying the term with γ in front of the sign of the integral in eq. (30), then collecting terms by powers of \hbar in the last multiplier of this equation and retaining only terms of order \hbar , we obtain the two-loop contribution to the effective action of the theory :

$$\Gamma'(G, \mathcal{D}) = 4\gamma^4(N^2 - 1)V - i\hbar \ln \int D\Phi \hbar^4 e^{S^q} \left[1 + \hbar \left(S^{(\hbar)} + \frac{1}{2}(S^{(\sqrt{\hbar})})^2 \right) \right]. \quad (34)$$

Expanding in \hbar we obtain

$$\Gamma'(G, \mathcal{D}) = 4\gamma^4(N^2 - 1)V = -i\hbar \ln \int D\Phi \hbar^4 e^{S^q} - i\hbar^2 \left\langle S^{(\hbar)} + \frac{1}{2}(S^{(\sqrt{\hbar})})^2 \right\rangle, \quad (35)$$

The first term is the result of integration e^{S^q} on all 8 fields (see the notation (10)), together with the notations (31) and (28). It gives, omitting unimportant constants $-i\hbar \ln \hbar^4$ and $\frac{i}{2}\hbar Tr \ln [\partial_\mu \partial_\nu]$:

$$-i\hbar \ln \int D\Phi \hbar^4 e^{S^q} = -i\hbar \ln [Det(\mathcal{D}^{-1})^{ab}]. \quad (36)$$

Note that integration over the fields $\varphi, \bar{\varphi}$ and $\omega, \bar{\omega}$ gives unity as well as over A and λ .

The second term of (35) is a sum of all vacuum expectation values of all the field configurations, included inside brackets $\langle \rangle$. One has to write down all these averages explicitly, making possible use of the Wick theorem for the computing every vacuum expectation value. Thus, all configurations with non-zero expectation values, which contribute to the second term of eq. (35) are written as follows:

$$\begin{aligned} \left\langle S^{(\hbar)} + \frac{1}{2}(S^{(\sqrt{\hbar})})^2 \right\rangle &= -\frac{i}{4}g^2 f^{abc} f^{apr} \langle A_\mu^b A_\nu^c A_\mu^p A_\nu^r \rangle - \\ &-i \frac{\delta \Gamma_2(G)}{\delta G_{\mu\nu}^{ab}} \langle A_\mu^a A_\nu^b \rangle + i \frac{\delta \Gamma_2(\mathcal{D})}{\delta \mathcal{D}^{ba}} \langle \bar{C}^a C^b \rangle - \\ &- \frac{1}{2}g^2 f^{abc} f^{prs} \langle (\partial_\mu A_\nu^a) A_\mu^b A_\nu^c (\partial_\rho A_\sigma^p) A_\rho^r A_\sigma^s \rangle - \\ &- \frac{1}{2}g^2 f^{acb} f^{prs} \langle \bar{\Theta}^a (\partial_\mu A_\mu^c \Theta^b) \bar{\Theta}^p (\partial_\nu A_\nu^r \Theta^s) \rangle. \quad (37) \end{aligned}$$

Let us consider these expectation values, making use of the Wick theorem. Here we write down only the result of the computations.

$$-\frac{1}{2}g^2 f^{abc} f^{prs} \langle \partial_\mu^{(x)} A_\mu^a(x) A_\mu^b(x) A_\nu^c(x) \partial_\rho^{(y)} A_\sigma^p(y) A_\rho^r(y) A_\sigma^s(y) \rangle =$$

$$\begin{aligned} &= \frac{1}{2}g^2 (f^{abc})^2 \{ -G_{\nu\rho}(x, y) (\partial_\mu^{(x)} G_{\nu\sigma}(x, y)) (\partial_\rho^{(y)} G_{\mu\sigma}(x, y)) + \\ &+ G_{\mu\rho}(x, y) (\partial_\mu^{(x)} G_{\nu\sigma}(x, y)) (\partial_\rho^{(y)} G_{\nu\sigma}(x, y)) \}. \quad (38) \end{aligned}$$

For four gluon fields at the same space-time point $A_\mu^a(x)$, one gets:

$$\begin{aligned} \frac{i}{4}g^2 f^{abc} f^{apr} \langle A_\mu^b(x) A_\nu^c(x) A_\mu^p(x) A_\nu^r(x) \rangle &= \frac{i}{4}g^2 (f^{abc})^2 \times \\ &\times \{ G_{\mu\nu}(x, x) G_{\mu\nu}(x, x) - G_{\mu\mu}(x, x) G_{\nu\nu}(x, x) \}. \quad (39) \end{aligned}$$

For anti-commuting ghosts (the first three fields are located in the same space-time point, say x , and let y be space-time point for the location of the last three fields), one gets:

$$\begin{aligned} -\frac{1}{2}g^2 f^{acb} f^{prs} \langle (\partial_\mu^{(x)} \bar{C}^a) A_\mu^c C^b|_{(x)} (\partial_\nu^{(y)} \bar{C}^p) A_\nu^r C^s|_{(y)} \rangle &= \\ = -\frac{1}{2}g^2 (f^{acb})^2 \{ \partial_\mu^{(x)} \mathcal{D}(y, x) \partial_\nu^{(y)} \mathcal{D}(x, y) G_{\mu\nu}(x, y) \}. \quad (40) \end{aligned}$$

Now with a close analogy to the ghosts, for Grassmann fields $\omega, \bar{\omega}$ we get

$$\begin{aligned} \frac{1}{2}g^2 f^{acb} f^{prs} \langle (\partial_\mu^{(x)} \bar{\omega}_i^a) A_\mu^c \omega_i^b|_{(x)} (\partial_\nu^{(y)} \bar{\omega}_j^p) A_\nu^r \omega_j^s|_{(y)} \rangle &= \\ = \frac{1}{2}g^2 (f^{acb})^2 \{ \partial_\mu^{(x)} G_{ji}^\omega(y, x) \partial_\nu^{(y)} G_{ij}^\omega(x, y) G_{\mu\nu}(x, y) \}. \quad (41) \end{aligned}$$

Here $G_{ij}^\omega(x, y)$ is a tree level propagator for the fields $\omega, \bar{\omega}$ and indices i, j take on $d(N^2 - 1)$ values. For a pair of complex conjugate bosonic fields $\varphi, \bar{\varphi}$ we get

$$\begin{aligned} \frac{1}{2}g^2 f^{acb} f^{prs} \langle (\partial_\mu^{(x)} \bar{\varphi}_i^a) A_\mu^c \varphi_i^b|_{(x)} (\partial_\nu^{(y)} \bar{\varphi}_j^p) A_\nu^r \varphi_j^s|_{(y)} \rangle &= \\ = -\frac{1}{2}g^2 (f^{acb})^2 \{ \partial_\mu^{(x)} G_{ij}^\varphi(x, y) \partial_\nu^{(y)} G_{ij}^\varphi(x, y) G_{\mu\nu}(x, y) \}, \quad (42) \end{aligned}$$

where $G_{ij}^\varphi(x, y)$ is a tree level propagator for the fields $\varphi, \bar{\varphi}$ and indices i, j are the same ones as in the previous case.

Finally, for the quantity $\Gamma'(G, \mathcal{D})$ from expressions (35) - (42) one has:

$$\Gamma'(G, \mathcal{D}) = \frac{i}{2}\hbar Tr 1 + 4\gamma^4(N^2 - 1)V - i\hbar Tr \ln(\mathcal{D}^{-1})^{ab} -$$

$$-\hbar^2 \{ G_{\mu\nu}^{ab} \frac{\delta \Gamma_2^{(1)}(G)}{\delta G_{\mu\nu}^{ab}} + \mathcal{D}^{ba} \frac{\delta \Gamma_2^{(1)}(\mathcal{D})}{\delta \mathcal{D}^{ba}} + \frac{i}{4}g^2 (f^{abc})^2 \times$$

$$\begin{aligned}
& \times (G_{\mu\nu}G_{\mu\nu} - G_{\mu\mu}G_{\nu\nu}) + \frac{i}{2}g^2(f^{acb})^2[-\partial_\mu^{(x)}\mathcal{D}\partial_\nu^{(y)}\mathcal{D}G_{\mu\nu} - \\
& -\partial_\mu^{(x)}G_{ji}^\omega\partial_\nu^{(y)}G_{ij}^\omega G_{\mu\nu} + \partial_\mu^{(x)}G_{ij}^\varphi\partial_\nu^{(y)}G_{ij}^\varphi G_{\mu\nu}] + \frac{i}{2}g^2(f^{abc})^2 \times \\
& \times [-G_{\nu\rho}(\partial_\mu G_{\nu\sigma})(\partial_\rho G_{\mu\sigma}) + G_{\mu\rho}(\partial_\mu G_{\nu\sigma})(\partial_\rho G_{\nu\sigma})]. \quad (43)
\end{aligned}$$

As it follows from the explicit definition of the propagators G_{ij}^ω and G_{ij}^φ (it can be found during the explicit integration of the expression (36) for S^q with the notations (31) and (28)), $G_{ij}^\omega \equiv G_{ij}^\varphi$. Inasmuch as these propagators enter the eq. (43) via terms of equivalent form, their whole contribution cancels:

$$-\partial_\mu^{(x)}G_{ij}^\omega\partial_\nu^{(y)}G_{ij}^\omega G_{\mu\nu} + \partial_\mu^{(x)}G_{ij}^\varphi\partial_\nu^{(y)}G_{ij}^\varphi G_{\mu\nu} = 0. \quad (44)$$

Thus, the effective action has no dependence on auxiliary unphysical fields $\varphi, \bar{\varphi}$ and $\omega, \bar{\omega}$.

Finally, taking into account that for the gauge group $SU(N)$:

$$f^{ade}f^{bde} = N\delta^{ab}, \quad (45)$$

the explicit expression (22) for the effective action of the theory reads as:

$$\begin{aligned}
\Gamma(G, \mathcal{D}) &= 4\gamma^4(N^2-1)V + \frac{1}{2}i\hbar Tr[\ln(G^{-1})_{\mu\nu}^{ab} + \\
& + (\tilde{D}_F^{-1})_{\mu\nu}^{ab}G_{\mu\nu}^{ab} - 1] - i\hbar Tr[\ln(\mathcal{D}^{-1})^{ab} + (S_F^{-1})^{ab}\mathcal{D}^{ba} - 1] - \\
& - \frac{1}{2}i\hbar^2 g^2 N \delta^{cc} \left\{ \frac{1}{2}(G_{\mu\nu}G_{\mu\nu} - G_{\mu\mu}G_{\nu\nu}) - \partial_\mu^{(x)}\mathcal{D}\partial_\nu^{(y)}\mathcal{D}G_{\mu\nu} - \right. \\
& \left. - G_{\nu\rho}(\partial_\mu G_{\nu\sigma})(\partial_\rho G_{\mu\sigma}) + G_{\mu\rho}(\partial_\mu G_{\nu\sigma})(\partial_\rho G_{\nu\sigma}) \right\}. \quad (46)
\end{aligned}$$

It is convenient to rewrite the expression (46) in momentum space, using the Fourier-transformed propagators, defined as follows:

$$G(p) = \int d^4x e^{\frac{i}{\hbar}p(x-y)} G(x-y); \quad (47)$$

(Inasmuch as we seek for the ground state, we consider the case of translation-invariant solutions, i.e., we take $G(x, y)$ to be a function only of $x - y$, and similarly for the other propagators.)

Passing to momentum space, for the eq. (46), divided by the volume V , we get

$$\begin{aligned}
& \frac{1}{V}\tilde{\Gamma}(G, \mathcal{D}) = 4\gamma^4(N^2-1) + \\
& + \frac{1}{2}i\hbar \int \frac{d^4p}{(2\pi\hbar)^4} \left(Tr \ln G^{-1}(p) + Tr \tilde{D}_F^{-1}(p)G(p) - 1 \right) -
\end{aligned}$$

$$\begin{aligned}
& - i\hbar \int \frac{d^4p}{(2\pi\hbar)^4} (Tr \ln \mathcal{D}^{-1}(p) + Tr S_F^{-1}(p)\mathcal{D}(p) - 1) - \\
& - \frac{i}{4}\hbar^2 g^2 N \delta^{cc} \int \frac{d^4p d^4r}{(2\pi\hbar)^8} \{ G_{\mu\nu}(p)G_{\mu\nu}(r) - G_{\mu\mu}(p)G_{\nu\nu}(r) \} - \\
& - \frac{i}{2}g^2 N \delta^{cc} \int \frac{d^4p d^4r}{(2\pi\hbar)^8} \{ \mathcal{D}(p)\mathcal{D}(r)G_{\mu\nu}(p-r)p_\mu r_\nu + \\
& + G_{\nu\rho}(p)G_{\nu\sigma}(p+r)G_{\mu\sigma}(r)(p+r)_\mu r_\rho - \\
& - G_{\mu\rho}(p)G_{\nu\sigma}(p+r)G_{\nu\sigma}(r)(p+r)_\mu r_\rho \}. \quad (48)
\end{aligned}$$

Thus we have the two-loop generalized effective action, written in the momentum representation.

IV. SCHWINGER-DYSON EQUATIONS

In order to obtain a closed system of equations on the gluon propagator $G_{\mu\nu}^{ab}(p)$, the ghost propagator $\mathcal{D}^{ab}(p)$ and the Gribov parameter γ one has to take variational derivatives of the effective action (48) with respect to these parameters and equate each of them to zero (see eq. (6)).

Before doing this, it is worth to remember that in the momentum representation the expression (23) for the free boson propagator modified with respect to the Zwanziger term is given by

$$\tilde{D}_F^{-1}(p) = D_F^{-1}(p) + 2g^2\gamma^4 N \frac{i\hbar^2}{p^2}. \quad (49)$$

From here on we will put $\hbar = 1$.

Varying expression (48) with respect to γ , we get

$$\begin{aligned}
& \frac{1}{V} \frac{\delta \tilde{\Gamma}(G, \mathcal{D})}{\delta \gamma} = 0 = \\
& = 4\gamma^3 \left[4(N^2-1) - g^2 N \int \frac{d^4p}{(2\pi)^4} Tr \frac{G_{\mu\nu}^{ab}(p)}{p^2} \right], \quad (50)
\end{aligned}$$

and after rejecting the trivial solution $\gamma = 0$, it looks as

$$1 = \frac{Ng^2}{4(N^2-1)} \int \frac{d^4p}{(2\pi)^4} Tr \frac{G_{\mu\nu}^{ab}(p)}{p^2}. \quad (51)$$

Variation with respect to $\mathcal{D}^{ab}(q)$ is given by

$$\frac{1}{V} \frac{\delta \tilde{\Gamma}(G, \mathcal{D})}{\delta \mathcal{D}^{ab}(q)} = 0 = i(\mathcal{D}^{-1}(q) - S_F^{-1}(q))^{ab} +$$

$$\begin{aligned}
& -\frac{i}{2}g^2N\delta^{ab}\left[\int\frac{d^4r}{(2\pi)^4}\mathcal{D}(r)G_{\mu\nu}(q-r)q_\mu r_\nu +\right. \\
& \left. +\int\frac{d^4p}{(2\pi)^4}\mathcal{D}(p)G_{\mu\nu}(p-q)p_\mu q_\nu\right]. \quad (52)
\end{aligned}$$

Due to the property $G(r) = G(-r)$ this expression after changing of mute indices $p \rightarrow r$ and $\mu \rightarrow \nu$ in the last term, can be written in the form

$$\begin{aligned}
& (\mathcal{D}^{-1}(q))^{ab} = (S_F^{-1}(q))^{ab} + \\
& + Ng^2\delta^{ab}\int\frac{d^4r}{(2\pi)^4}\mathcal{D}(r)G_{\mu\nu}(q-r)q_\mu r_\nu. \quad (53)
\end{aligned}$$

Variation with respect to $G_{\mu\nu}^{ab}(q)$ is given by

$$\begin{aligned}
& \frac{1}{V}\frac{\delta\tilde{\Gamma}(G,\mathcal{D})}{\delta G_{\mu\nu}^{ab}(q)} = 0 = \frac{i}{2}\left(-G^{-1}(q) + \tilde{D}_F^{-1}(q)\right)_{\mu\nu}^{ab} - \frac{i}{2}g^2N\delta^{ab}\times \\
& \times \int\frac{d^4p}{(2\pi)^4}\mathcal{D}(p)\mathcal{D}(p-q)p_\mu(p-q)_\nu + (\text{gluon loops}). \quad (54)
\end{aligned}$$

Some simplifications can be done in the expressions above. In the eq. (51) we take the trace on color and Lorentz indices. In the Landau gauge the gluon propagator is transverse on vector indices:

$$G_{\mu\nu}^{ab}(p) = G(p^2)\mathcal{P}_{\mu\nu}(p)\delta^{ab}, \quad (55)$$

where

$$\mathcal{P}_{\mu\nu}(p) \equiv \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right] \quad (56)$$

is the transverse projector. In the eq. (53) we shift the integration variable according to $r \rightarrow r + q$ and use eq. (55). The similar shift of integration variable will be applied to the eq. (54).

Now we are ready to write down a closed system of equations of motion for the Gribov parameter, the ghost and gluon propagators:

$$1 = \frac{3}{4}Ng^2\int\frac{d^4p}{(2\pi)^4}\frac{G(p)}{p^2}; \quad (57)$$

$$(\mathcal{D}^{-1}(q))^{ab} = (S_F^{-1}(q))^{ab} +$$

$$+ Ng^2\delta^{ab}\int\frac{d^4r}{(2\pi)^4}\mathcal{D}(q+r)G(r^2)\left[\frac{r^2q^2 - (qr)^2}{r^2}\right]; \quad (58)$$

$$(G^{-1}(q))_{\mu\nu}^{ab} = \left(\tilde{D}_F^{-1}(q)\right)_{\mu\nu}^{ab} - g^2N\delta^{ab}\times$$

$$\times \int\frac{d^4p}{(2\pi)^4}\{\mathcal{D}(p+q)\mathcal{D}(p)(p+q)_\mu p_\nu\} + (\text{gluon loops}). \quad (59)$$

It is worth noticing, that the Gribov parameter γ does not enter its equation of motion (57) explicitly, but through the modified free gluon propagator $\tilde{D}_F(q)$ (according to (49)), which is present in equation (59) for the gluon propagator.

We wish to determine the asymptotic form of the propagators at low momentum. For this purpose we let the external momentum in the SDEs written above be asymptotically small. In this case the loop integration will be dominated by asymptotically small loop momentum, so the propagators inside the integrals may also be replaced by their asymptotic values. As one can see from Section 2, where it is argued that the ghost propagator should be more singular and the gluon less singular than a simple pole, gluon loops integration at low momentum in eq. (59) may be neglected.

Let us consider the eq. (58). One of the form of the horizon conditions, that guaranties the absence of the Gribov copies, is written as [16]

$$\lim_{q \rightarrow 0} [q^2 \mathcal{D}(q^2)]^{-1} = 0.$$

To impose this condition, we divide eq. (58) by q^2 , and obtain (after the factorization of color indices)

$$0 = i + Ng^2\int\frac{d^4r}{(2\pi)^4}\mathcal{D}(r^2)G(r^2)[1 - \cos(\hat{q}\hat{r})]. \quad (60)$$

Subtracting this equation from the previous one (58), we get

$$\begin{aligned}
& \mathcal{D}^{-1}(q^2) = -Ng^2\int\frac{d^4r}{(2\pi)^4}G(r^2)[\mathcal{D}(r^2) - \\
& - \mathcal{D}((q+r)^2)]\left[\frac{r^2q^2 - (qr)^2}{r^2}\right]. \quad (61)
\end{aligned}$$

We now turn to the SDE for the gluon propagator (59) with comments below it. We apply the transverse projector and take the trace on color and Lorentz indices and obtain taking into account eq. (49):

$$G^{-1}(q^2) = iq^2 + 2Ng^2\gamma^4\frac{i}{q^2} -$$

$$- \frac{Ng^2}{3}\int\frac{d^4p}{(2\pi)^4}\mathcal{D}((p+q)^2)\mathcal{D}(p^2)\left[\frac{p^2q^2 - (pq)^2}{q^2}\right]. \quad (62)$$

Notice, that in the limit $q \rightarrow 0$ the first term vanishes.

In order to simplify further analysis and compare our results with other authors, we rewrite SDEs for the gluon and ghost propagators in Euclidean space:

$$\mathcal{D}^{-1}(q^2) = Ng^2 \int \frac{d^4 p}{(2\pi)^4} G(p^2) [\mathcal{D}(p^2) - \mathcal{D}((q+p)^2)] \times \left[\frac{p^2 q^2 - (pq)^2}{p^2} \right]. \quad (63)$$

$$G^{-1}(q^2) = 2g^2 \gamma^4 N \frac{1}{q^2} + \frac{Ng^2}{3} \int \frac{d^4 p}{(2\pi)^4} \mathcal{D}((p+q)^2) \mathcal{D}(p^2) \times \left[\frac{p^2 q^2 - (pq)^2}{q^2} \right]. \quad (64)$$

This system of SDEs for the gluon and ghost propagators in Landau gauge, written in Euclidean space, will be analyzed in next sections.

V. ZERO-TH-ORDER ANALYSIS OF SDES

Now we consider the first approximation which corresponds to retaining only the first term in the right-hand side of (59) without any changes in other equations. It is in fact the zeroth-order approximation. In eq. (59) the contribution at this order gives

$$\left(G^{(1)}(q) \right)_{\mu\nu}^{ab} = \frac{q^2}{q^4 + 2Ng^2 \gamma^4} \delta^{ab} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right). \quad (65)$$

To see this one has to inverse left-hand side and the first term in the right-hand side of eq. (59) and rewrite the result in Euclidean space.

After substitution of this approximation to the Euclidean form of eq. (57), we get

$$1 = \frac{3}{4} Ng^2 \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^4 + 2g^2 N \gamma^4}. \quad (66)$$

Note, that the gluon propagator (65) and the gap equation (66) were first derived by Gribov [1]. Furthermore, so-called horizon condition (66) was obtained by making use of the zeroth-order approximation (65), so we see that SDE for the Gribov parameter (57) defines full horizon condition.

We now turn to the ghost SDE and apply zeroth-order approximation (65) to it. For this purpose it is more convenient to use Euclidean form of eq. (53). Substituting the explicit expression for $G_{\mu\nu}(q-r)$ (according to (65)) in the horizon condition (66) one gets

$$\mathcal{D}^{-1}(q) = Ng^2 q_\mu q_\nu \int \frac{d^4 r}{(2\pi)^4} \frac{1}{r^4 + \lambda^4} \left(g_{\mu\nu} - \frac{r_\mu r_\nu}{r^2} \right) \times [1 - r^2 \mathcal{D}(q-r)], \quad (67)$$

with $\lambda^4 \equiv 2Ng^2 \gamma^4$.

This is a nonlinear integral equation for the \mathcal{D} -function and we now have to solve it. One of the possible ways to do this is iteration scheme with a free ghost propagator $S_F(q) = 1/q^2$ as a zeroth-order approximation $\mathcal{D}_{(0)}(q)$ of $\mathcal{D}(q)$. In other words, in order to obtain first-order approximation $\mathcal{D}_{(1)}^{-1}(q)$ of $\mathcal{D}^{-1}(q)$, we have to replace $\mathcal{D}(q-r) \rightarrow \mathcal{D}_{(0)}(q-r) \equiv 1/(q-r)^2$ in the right-hand side of eq. (67). Doing this we get

$$\mathcal{D}_{(1)}^{-1}(q) = Ng^2 q_\mu q_\nu \int \frac{d^4 r}{(2\pi)^4} \frac{1}{r^4 + \lambda^4} \left(g_{\mu\nu} - \frac{r_\mu r_\nu}{r^2} \right) \frac{q^2 - 2qr}{(q-r)^2}. \quad (68)$$

Notice, that exactly the same expression was used by Gribov in order to obtain the infrared behaviour of the ghost propagator [1].

We emphasize that while Gribov in fact solved the equation for the ghost propagator approximately, we derive some integral equation and we look for the exact solutions to it.

After integration of the right-hand side of (68) for the first-order approximation of $\mathcal{D}(q)$ one gets [17]:

$$\mathcal{D}_{(1)}(q) = \frac{128\pi\lambda^2}{3g^2 N} \frac{1}{q^4}. \quad (69)$$

Notice, that further making use of such iterational scheme for the eq. (67) results in incompatible system of equations. Thus the natural question appears: is the iterational procedure, used for solving the equation (67), admissible? The obvious answer is "no". Because this iteration procedure is not converging. Consequently, perturbation theory does not work and the result, obtained above, is in general incorrect, though it gives qualitatively right behaviour. In order to solve the equation (67) correctly some another method is needed.

Let us try to solve the equation for the ghost propagator (67), written in the form, similar to (63) (with the explicit gluon propagator (65)) in the infrared:

$$[g\mathcal{D}(q^2)]^{-1} = Ng \int \frac{d^4 r}{(2\pi)^4} [\mathcal{D}(r^2) - \mathcal{D}((q+r)^2)] \times \left[\frac{r^2 q^2 - (qr)^2}{r^4 + \lambda^4} \right]. \quad (70)$$

In order to solve this equation, one can seek for the solution in the form

$$g\mathcal{D}(q^2) = \frac{A}{(q^2)^{1+\alpha}}. \quad (71)$$

Here A is some coefficient and α is called infrared critical exponent. After evaluation of the integrals, one gets

$$A^2 = \frac{32\pi^2\lambda^4}{N}. \quad (72)$$

Thus, the exact solution of the equation (67), which describes the infrared behaviour of the ghost propagator is

$$\mathcal{D}(q^2) = 4\sqrt{2} \frac{\pi\lambda^2}{g\sqrt{N}} \frac{1}{q^4}. \quad (73)$$

Notice, that the previously obtained solution (69) qualitatively coincides with ours, but has another coefficient. Thus our zeroth-order analysis qualitatively confirmed that in the infrared region the gluon propagator is suppressed (65), while the ghost propagator is enhanced (73).

VI. FULL ANALYSIS OF THE DERIVED ONE-LOOP SDES

We now turn to the SDEs (63) and (64). One can solve these equations using "weak-angle-dependence" approximation for the ghosts:

$$\mathcal{D}((q+p)^2) \approx \mathcal{D}(q^2)\Theta(q^2-p^2) + \mathcal{D}(p^2)\Theta(p^2-q^2). \quad (74)$$

Here $\Theta(x)$ is the ordinary step function. Substituting this approximate expression to the ghost SDE (63) and gluon SDE (64) and then carrying out the angles integration, one gets

$$\mathcal{D}^{-1}(q^2) = Aq^2 \int_0^{q^2} dp^2 p^2 G(p^2) [\mathcal{D}(p^2) - \mathcal{D}(q^2)], \quad (75)$$

$$G^{-1}(q^2) = \frac{\lambda^4}{q^2} + B \left[\int_0^{q^2} dp^2 p^4 \mathcal{D}(p^2) \mathcal{D}(q^2) + \int_{q^2}^{\infty} dp^2 p^4 \mathcal{D}^2(p^2) \right], \quad (76)$$

with

$$A \equiv \frac{3Ng^2}{16(2\pi)^2}, \quad B \equiv \frac{Ng^2}{16(2\pi)^2}, \quad \lambda^4 \equiv 2Ng^2\gamma^4. \quad (77)$$

Solutions may be found in the form of power-laws:

$$G(q^2) = C_G(q^2)^{-\alpha}, \quad (78)$$

$$\mathcal{D}(q^2) = C_D(q^2)^{-\beta}, \quad (79)$$

with C_G and C_D – dimensionless constants, α and β – some exponents. It turned out, that straightforward substitution of ansatz (78), (79) in eqs. (75), (76) suffers from certain mathematical difficulties and it is more

convenient to do first some transformations in the equations above. We divide equation (75) by q^2 then carry out differentiation with respect to q^2 in derived equation and in eq. (76), and obtain:

$$\frac{1}{q^4} \mathcal{D}^{-1}(q^2) + \frac{1}{q^2} \mathcal{D}^{-2}(q^2) \mathcal{D}'(q^2) = A \mathcal{D}'(q^2) \int_0^{q^2} dp^2 p^2 G(p^2). \quad (80)$$

$$G^{-2}(q^2) G'(q^2) = \frac{\lambda^4}{q^4} - B \mathcal{D}'(q^2) \int_0^{q^2} dp^2 p^4 \mathcal{D}(p^2). \quad (81)$$

Now the ansatz (78), (79) can be applied. We substitute this power-laws into our equations and carry out all the integrations (under the assumption of $\beta \neq 3$ and $\alpha \neq 2$). Then we get

$$(-\beta+1)C_D^{-1}(q^2)^{\beta-2} + \frac{\beta AC_D C_G}{-\alpha+2} (q^2)^{-\beta-\alpha+1} = 0; \quad (82)$$

$$-\alpha C_G^{-1}(q^2)^{\alpha-1} - \lambda^4 (q^2)^{-2} - \frac{\beta B C_D^2}{-\beta+3} (q^2)^{-2\beta+2} = 0. \quad (83)$$

We equate exponents of q^2 in both sides of this equations and obtain following equations for the parameters α and β :

$$\beta - 2 = -\beta - \alpha + 1 \quad (84)$$

$$\alpha - 1 = -2 = -2\beta + 2, \quad (85)$$

which give us a unique solution

$$\alpha = -1 \quad \beta = 2. \quad (86)$$

In terms of the infrared critical exponents

$$\alpha_G = \alpha - 1, \quad \alpha_D = \beta - 1, \quad (87)$$

(see (3), (4)), one has

$$\alpha_G = -2, \quad \alpha_D = 1. \quad (88)$$

We now turn to the equations (82) and (83) in order to obtain the coefficients C_G and C_D . Thus we get

$$-C_D^{-1} + \frac{2}{3} AC_D C_G = 0; \quad (89)$$

$$C_G^{-1} - \lambda^4 - 2BC_D^2 = 0. \quad (90)$$

Along with the notations (77) this gives

$$C_G = \frac{1}{\lambda^4} [1 + 3BA^{-1}] = \frac{2}{\lambda^4}; \quad (91)$$

$$C_D = \sqrt{\frac{3}{2} A^{-1} C_G^{-1}} = 4 \frac{\pi\lambda^2}{g\sqrt{N}}, \quad (92)$$

with $\lambda^4 \equiv 2Ng^2\gamma^4$, where γ is the Gribov parameter.

Consequently, the solution of the SDEs (63) and (64) is the following infrared behaviour of the gluon and ghost propagators, respectively:

$$G(q^2) = \frac{2}{\lambda^4} q^2, \quad (93)$$

$$\mathcal{D}(q^2) = 4 \frac{\pi \lambda^2}{g \sqrt{N}} \frac{1}{q^4}. \quad (94)$$

Thus one can see that the one-loop results are in good qualitative agreement with the tree-level ones. The coefficients (91), (92) obtained at this order, differ from the zeroth-order ones (65), (73) by numerical multiplier and, hopefully, are more precise.

VII. DISCUSSION AND CONCLUSIONS

Formalism of the generalized effective action with implemented restriction to the Gribov horizon was used for the investigating of the infrared behaviour of the gluon and ghost propagators. The system of the SDEs on the gluon propagator (59), the ghost propagator (58) and the Gribov parameter (57) is derived in this approach. These equations are obtained using two-loop approximation for the generalized effective action.

The SDE for the Gribov parameter is in fact the full horizon condition, which reduces to the Gribov's one at zeroth order. Furthermore, for the gluon propagator at

this order follows the same expression, as first pointed out by Gribov. Expression for the ghost propagator, following from zeroth-order approximation, confirms the qualitative behaviour but differs from the Gribov's one by some coefficient.

Obtained SDEs are solved in the first-order approximation as well. Infrared critical exponents and coefficients for the gluon and ghost propagators are obtained. Thus, we derive the expressions, which describe the infrared behaviour of the gluon (93) and ghost (94) propagators. A solution for the infrared exponents was obtained before [7], whereas the coefficients for the propagators at this order are derived firstly in present work.

Notice, that the first-order results agree with the zeroth-order ones with a different numerical coefficients at small momenta. For the gluon propagator this difference reduces to 1/2 and $\sqrt{2}$ for the ghost one.

Thus cutting off the measure of the functional integral at the (first) Gribov region leads to infrared enhancement of the ghost propagator and suppression of the gluon one. Such a behaviour of the propagators supports the picture of color confinement [1, 2, 14].

It would be interesting to study in the effective action formalism of CJT the possibility of generating the gluon mass [20] when the Zwanziger horizon condition is implemented [18].

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